Financial Contagion Model with Time Differences*

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1 Introduction

In securities market, the financial contagion problem is a serious issue. For example, a price change of a stock in one country could cause price changes of another countries whose fundamentals are independent of the country. In addition, world-wide activities of mutual funds would accelerates these phenomena. In recent months, for example, the U. S. subprime loan problem has brought on a crisis with a significant impact, and many countries have taken severe monetary policies for several years. Although the issue is serious and the countermeasures must be implemented as soon as possible, no basic tool for theoretical analysis of the impact is fully developed yet.

In this paper, we develop a three-period model of multi-asset trading based on Caballé and Krishnan (1994):

1. There are three types of market participants: informed, imperfectly competitive speculators (classified to two type: international and domestic), uninformed, competitive market makers, and liquidity traders,
2. there are idiosyncratic and systematic sources of risk that affect the dividend of the traded securities, and
3. each trade occurs at the end of each period.

There are market impacts in the sense that speculators are imperfectly competitive. They have incentives to exploit their payoffs by using their information. As a result, the security prices between two countries that are fundamentally unrelated are correlated.

2 The Model

There are three countries whose trading hours are not overlapped. Each country has one risky asset respectively and there is one riskless asset on a global basis. After all trades, a trader who has one lot of risky assets receives dividends, a 3×1 multivariate normally distributed random vector v.

$$v=u+\beta\theta, \tag{1}$$

where u is a 3×1 random vector of idiosyncratic shocks, θ is a 2×1 random vector of systematic sources of risk, and β is a 3×2 matrix of factor loadings:

$$\beta=\begin{pmatrix} 1 & 0 \\ 0.5 & 0.5 \\ 0 & 1 \end{pmatrix}. \tag{2}$$

We assume that u and θ are independent, $u \sim N(\bar{u}, \Sigma_u)$, $\theta \sim N(\bar{\theta}, \Sigma_\theta)$, and Σ_u and Σ_θ are diagonal and nonsingular.

There are five types of market participants: competitive market makers, K_I international traders, $K_t (t=1, 2, 3)$ domestic traders, and a liquidity trader. All traders are risk neutral.

The international traders and the domestic traders observe their private signals about u and θ at $t=0$:

$$S_{uk}=u+\varepsilon_{uk},\ \varepsilon_{uk} \sim MND(\underline{0}, \Sigma_{\varepsilon u}), \tag{3}$$

$$S_{\theta k}=\theta+\varepsilon_{\theta k},\ \varepsilon_{\theta k} \sim NND(\underline{0}, \Sigma_{\varepsilon \theta}). \tag{4}$$

u, θ, ε_{uk}, and $\varepsilon_{\theta k} (k=1, 2, \cdots, K_1+K_2+K_3+K_I)$ are independent. $\Sigma_{\varepsilon u}$ and $\Sigma_{\varepsilon \theta}$ are diagonal. The term k here is assigned in the order of the international traders, the domestic traders in the country 1, 2, and 3 in sequence. In the following, "a trader k_I" represents an international trader, and "a trader $k_t (t=1, 2, 3)$" represents a domestic trader in the country t.

The trade in the country $t\ (t=1, 2, 3)$ occurs at the period t. The international traders can place their orders at all periods. The domestic traders k_t can place their orders only at the period t. Each trader $k\ (k=k_I, k_t)$ places his market order x_{kt} without observing the orders of other traders at the period t. The liquidity trader places his market order that follows an independent normal distribution at each period t. Formally, the order of the liquidity trader is written by the a 3×1 random vector:

$$z \sim MND(\bar{z}, \Sigma_z), \tag{5}$$

where $z(t)$ represents the order of the liquidity trader at period t, and Σ_z is diagonal.

The market makers have no private information. They observe the net order that all traders place and determine the price p_t. Since they are competitive, they put p_t to make equal to the expected value of the risky asset of the country t, conditional on their information that they have observed so far.

At $t=4$, u and θ are realized, and traders obtain their dividends. Every trader k has NAV_{4k} units of riskless asset at $t=0$, and maximizes his utility

$$U_k=U(NAV_{4k})=NAV_{0k}+x_{k1}(\nu(1)-p_1)+x_{k2}(\nu(2)-p_2)+x_{k3}(\nu(3)-p_3). \tag{6}$$

We define some notations. $E_t^k(\cdot)$ and $E_t^M(\cdot)$ are expected values conditional on events that a trader k and a market maker can observe before their decision making at period t respectively.

3 Linear Equilibrium

We limit our investigation to a linear equilibrium. In the equilibrium, in each period $t=1, 2, 3$

1. every trader k maximizes his utility:

$$\max_{x_{kt}} E_t^k(U_k),$$

2. the market makers place the price to equal to the expected value of the risky asset of the country t:

$$p_t(\omega_t, \{p_u\}_{u<t})=E_t^M(v).$$

We assume actions of all traders are linear with respect to their information. That is,

$$p_1(\omega_1)=a_{01}+a_{11}(\omega_1-\bar{\omega}_1), \tag{7}$$

$$p_2(p_1, \omega_2)=a_{02}+a_{12}(p_1-\bar{p}_1)+a_{22}(\omega_2-\bar{\omega}_2), \tag{8}$$

$$p_3(p_1, p_2, \omega_3)=a_{03}+a_{13}(p_1-\bar{p}_1)+a_{23}(p_2-\bar{p}_2+a_{33}(\omega_3-\bar{\omega}_3), \tag{9}$$

$$x_{k1}(S_{uk}, S_{\theta k})=b_{k01}+b'_{ku1}(S_{uk}-\bar{u})+b'_{k\theta 1}(S_{\theta k}-\bar{\theta}), \tag{10}$$

$$x_{k2}(p_1, S_{uk}, S_{\theta k})=b_{k02}+b_{k12}(p_1-\bar{p}_1)+b'_{ku2}(S_{uk}-\bar{u})+b'_{k\theta 2}(S_{\theta k}-\bar{\theta}), \tag{11}$$

$$x_{k3}(p_1, p_2, S_{uk}, S_{\theta k})=b_{k03}+b_{k13}(p_1-\bar{p}_1)+b_{k23}(p_2-\bar{p}_2) \tag{12}$$

$$+b'_{ku3}(S_{uk}-\bar{u})+b'_{k03}(S_{\theta k}-\bar{\theta}),$$

where

$$\bar{p}_1=a_{01}, \tag{13}$$

$$\bar{p}_2=a_{02}, \tag{14}$$

$$\bar{\omega}_1=K_I b_{k_I 01}+K_1 b_{k_1 01}+\bar{z}(1), \tag{15}$$

$$\bar{\omega}_2=K_I b_{k_I 02}+K_2 b_{k_2 02}+\bar{z}(2), \tag{16}$$

$$\bar{\omega}_3=K_I b_{k_I 03}+K_3 b_{k_3 03}+\bar{z}(3). \tag{17}$$

4 Information

4.1 Ex-ante Distributions

$$cov(v(1), \omega_1)=(K_I b_{k_I u1}(1)+K_1 b_{k_1 u1}(1))\Sigma_u(1, 1) \tag{18}$$

$$+(K_I b_{k_I \theta 1}(1)+K_1 b_{k_1 \theta 1}(1))\Sigma_\theta(1, 1),$$

$$\begin{aligned} cov(v(2),p_1)=&\,a_{11}\{(K_I b_{k_I u1}(2)+K_1 b_{k_1 u1}(2))\textstyle\sum_u(2,2)\\ &+\frac{1}{2}(K_I b_{k_I \theta 1}(1)+K_1 b_{k_1 \theta 1}(1))\textstyle\sum_\theta(1,1)\\ &+\frac{1}{2}(K_I b_{k_I \theta 1}(2)+K_1 b_{k_1 \theta 1}(2))\textstyle\sum_\theta(2,2)\}, \end{aligned} \tag{19}$$

$$\begin{aligned} cov(v(2),\omega_2)=&\,(K_I b_{k_I 12}+K_2 b_{k_2 12})cov(v(2),p_1)\\ &+(K_I b_{k_I u2}(2)+K_2 b_{k_2 u2}(2))\textstyle\sum_u(2,2)\\ &+\frac{1}{2}(K_I b_{k_I \theta 2}(1)+K_2 b_{k_2 \theta 2}(1))\textstyle\sum_\theta(1,1)\\ &+\frac{1}{2}(K_I b_{k_I \theta 2}(2)+K_2 b_{k_2 \theta 2}(2))\textstyle\sum_\theta(2,2), \end{aligned} \tag{20}$$

$$\begin{aligned} cov(v(3),p_1)=&\,a_{11}\{(K_I b_{k_I u1}(3)+K_1 b_{k_1 u1}(3))\textstyle\sum_u(3,3)\\ &+(K_I b_{k_I \theta 1}(2)+K_1 b_{k_1 \theta 1}(2))\textstyle\sum_\theta(2,2)\}, \end{aligned} \tag{21}$$

$$\begin{aligned} cov(v(3),p_2)=&\,a_{12}cov(v(3),p_1)\\ &+a_{22}\{(K_I b_{k_I 12}+K_2 b_{k_2 12})\,cov(v(3),p_1)\\ &+(K_I b_{k_I u2}(3)+K_2 b_{k_2 u2}(3))\textstyle\sum_u(3,3)\\ &+(K_I b_{k_I \theta 2}(2)+K_2 b_{k_2 \theta 2}(2))\textstyle\sum_\theta(2,2)\}, \end{aligned} \tag{22}$$

$$\begin{aligned} cov(v(3),\omega_3)=&\,(K_I b_{k_I 13}+K_3 b_{k_3 13})\,cov(v(3),p_1)\\ &+(K_I b_{k_I 23}+K_3 b_{k_3 23})\,cov(v(3),p_2)\\ &+(K_I b_{k_I u3}(3)+K_3 b_{k_3 u3}(3))\textstyle\sum_u(3,3)\\ &+(K_I b_{k_I 03}(2)+K_3 b_{k_3 \theta 3}(2))\textstyle\sum_\theta(2,2), \end{aligned} \tag{23}$$

$$cov(S_{uk},\omega_1)=\textstyle\sum_u(K_I b_{k_I u1}+K_1 b_{k_1 u1})+I_{\{k=k_I,k_1\}}\sum_{\varepsilon_u} b_{ku1},(I_{\{\cdot\}}\text{ is an index function}) \tag{24}$$

$$cov(S_{\theta k},\omega_1)=\textstyle\sum_\theta(K_I b_{k_I \theta 1}+K_1 b_{k_1 \theta 1})+I_{\{k=k_I,k_1\}}\sum_{\varepsilon_\theta} b_{k\theta 1}, \tag{25}$$

$$cov(S_{uk},p_1)=a_{11}cov(S_{uk},\omega_1), \tag{26}$$

$$cov(S_{\theta k},p_1)=a_{11}cov(S_{\theta k},\omega_1), \tag{27}$$

$$\begin{aligned} cov(S_{uk},\omega_2)=&\,(K_I b_{k_I 12}+K_2 b_{k_2 12})cov(S_{uk},p_1)\\ &+\textstyle\sum_u(K_I b_{k_I u2}+K_2 b_{k_2 u2})+I_{\{k=k_I,k_2\}}\sum_{\varepsilon_u} b_{ku2}, \end{aligned} \tag{28}$$

$$\begin{aligned} cov(S_{\theta k},\omega_2)=&\,(K_I b_{k_I 12}+K_2 b_{k_2 12})\,cov(S_{\theta k},p_1)\\ &+\textstyle\sum_\theta(K_I b_{k_I \theta 2}+K_2 b_{k_2 \theta 2})+I_{\{k=k_I,k_2\}}\sum_{\varepsilon_\theta} b_{k\theta 2}, \end{aligned} \tag{29}$$

$$cov(S_{uk},p_2)=a_{12}cov(S_{uk},p_1)+a_{22}cov(S_{uk},\omega_2) \tag{30}$$

$$cov(S_{\theta k},p_2)=a_{12}cov(S_{\theta k},p_1)+a_{22}cov(S_{\theta k},\omega_2), \tag{31}$$

$$\begin{aligned} cov(S_{uk},\omega_3)=&\,(K_I b_{k_I 13}+K_3 b_{k_3 13})\,cov(S_{uk},p_1)\\ &+(K_I b_{k_I 23}+K_3 b_{k_3 23})cov(S_{uk},p_2)\\ &+\textstyle\sum_u(K_I b_{k_I u3}+K_3 b_{k_3 u3})+I_{\{k=k_I,k_3\}}\sum_{\varepsilon_u} b_{ku3}, \end{aligned} \tag{32}$$

$$cov(S_{\theta k},\ \omega_3)=(K_I b_{k_I 13}+K_3 b_{k_3 13})\,cov(S_{\theta k},\ p_1)$$
$$+(K_I b_{k_I 23}+K_3 b_{k_3 23})\,cov(S_{\theta k},\ p_2)$$
$$+\sum\nolimits_{\theta}(K_I b_{k_I \theta 3}+K_3 b_{k_3 \theta 3})+I_{\{k=k_I,\ k_3\}}\sum\nolimits_{\varepsilon_\theta} b_{k\theta 3}, \tag{33}$$

$$cov(S_{uk},\ p_3)=a_{13}cov(S_{uk},\ p_1)+a_{23}cov(S_{uk},\ p_2)+a_{33}cov(S_{uk},\ \omega_3) \tag{34}$$

$$\sum\nolimits_{\omega_1}=K_I b'_{k_I u1}(\sum\nolimits_u+\sum\nolimits_{\varepsilon_u})b_{k_I u1}+K_I(K_I-1)b'_{k_I u1}\sum\nolimits_u b_{k_I u1}$$
$$+K_1 b'_{k_1 u1}(\sum\nolimits_u+\sum\nolimits_{\varepsilon_u})b_{k_1 u1}+K_1(K_1-1)b'_{k_1 u1}\sum\nolimits_u b_{k_1 u1}$$
$$+2K_I K_1 b'_{k_1 u1}\sum\nolimits_u b_{k_I u1}$$
$$+K_I b'_{k_I \theta 1}(\sum\nolimits_\theta+\sum\nolimits_{\varepsilon_\theta})b_{k_I \theta 1}+K_I(K_I-1)b'_{k_I \theta 1}\sum\nolimits_\theta b_{k_I \theta 1}$$
$$+K_1 b'_{k_1 \theta 1}(\sum\nolimits_\theta+\sum\nolimits_{\varepsilon_\theta})b_{k_1 \theta 1}+K_1(K_1-1)b'_{k_1 \theta 1}\sum\nolimits_\theta b_{k_1 \theta 1}$$
$$+2K_I K_1 b'_{k_1 \theta 1}\sum\nolimits_u b_{k_I \theta 1}+\sum\nolimits_z(1,\ 1), \tag{35}$$

$$\sum\nolimits_{p_1}=a_{11}^2\sum\nolimits_{\omega_1}, \tag{36}$$

$$cov(p_1,\ \omega_2)=(K_I b_{k_I 12}+K_2 b_{k_2 12})\sum\nolimits_{p_1}$$
$$+K_I cov(S_{uk_I},\ p_1)' b_{k_I u2}+K_2 cov(S_{uk_2},\ p_1)' b_{k_2 u2}$$
$$+K_I cov(S_{\theta k_I},\ p_1)' b_{k_I \theta 2}+K_2 cov(S_{\theta k_2},\ p_1)' b_{k_2 \theta 2}, \tag{37}$$

$$cov(p_1,\ p_2)=a_{12}\sum\nolimits_{p_1}+a_{22}cov(p_1,\ \omega_2), \tag{38}$$

$$\sum\nolimits_{\omega_2}=(K_I b_{k_I 12}+K_2 b_{k_2 12})\{cov(p_1,\ \omega_2)$$
$$+K_I cov(S_{uk_I},\ p_1)' b_{k_I u2}+K_2 cov(S_{uk_2},\ p_1)' b_{k_2 u2}$$
$$+K_I cov(S_{\theta K_I},\ p_1)' b_{k_I \theta 2}+K_2 cov(S_{\theta k_2},\ p_1)' b_{k_2 \theta 2}\}$$
$$+K_I b'_{k_I u2}(\sum\nolimits_u+\sum\nolimits_{\varepsilon_u})b_{k_I u2}+K_I(K_I-1)b'_{k_I u2}\sum\nolimits_u b_{k_I u2}$$
$$+K_2 b'_{k_2 u2}(\sum\nolimits_u+\sum\nolimits_{\varepsilon_u})b_{k_2 u2}+K_2(K_2-1)b'_{k_2 u2}\sum\nolimits_u b_{k_2 u2}$$
$$+2K_I K_2 b'_{k_2 u2}\sum\nolimits_u b_{k_I u2}$$
$$+K_I b'_{k_I \theta 2}(\sum\nolimits_\theta+\sum\nolimits_{\varepsilon_\theta})b_{k_I \theta 2}+K_I(K_I-1)b'_{k_I \theta 2}\sum\nolimits_\theta b_{k_I \theta 2}$$
$$+K_2 b'_{k_2 \theta 2}(\sum\nolimits_\theta+\sum\nolimits_{\varepsilon_\theta})b_{k_2 \theta 2}+K_2(K_2-1)b'_{k_2 \theta 2}\sum\nolimits_\theta b_{k_2 \theta 2}$$
$$+2K_I K_2 b'_{k_2 \theta 2}\sum\nolimits_u b_{k_I \theta 2}+\sum\nolimits_z(2,\ 2), \tag{39}$$

$$\sum\nolimits_{p_2}=a_{12}cov(p_1,\ p_2)+a_{12}a_{22}cov(p_1,\ \omega_2)+a_{22}^2\sum\nolimits_{\omega_2} \tag{40}$$

$$cov(p_1,\ \omega_3)=(K_I b_{k_I 13}+K_3 b_{k_3 13})\sum\nolimits_{p_1}+(K_I b_{k_I 23}+K_3 b_{k_3 23})\,cov(p_1,\ p_2)$$
$$+K_I cov(S_{uk_I},\ p_1)' b_{k_I u3}+K_3 cov(S_{uk_3},\ p_1)' b_{k_3 u3}$$
$$+K_I cov(S_{\theta k_I},\ p_1)' b_{k_I \theta 3}+K_3 cov(S_{\theta k_3},\ p_1)' b_{k_3 \theta 3}, \tag{41}$$

$$cov(p_2,\ \omega_3)=(K_I b_{k_I 13}+K_3 b_{k_3 13})\,cov(p_1,\ p_2)+(K_I b_{k_I 23}+K_3 b_{k_3 23})\sum\nolimits_{p_2}$$
$$+K_I cov(S_{uk_I},\ p_2)' b_{k_I u3}+K_3 cov(S_{uk_3},\ p_2)' b_{k_3 u3}$$
$$+K_I cov(S_{\theta k_I},\ p_2)' b_{k_I \theta 3}+K_3 cov(S_{\theta k_3},\ p_2)' b_{k_3 \theta 3}, \tag{42}$$

$$cov\,(p_1,\ p_3)=a_{13}\Sigma_{p_1}+a_{23}cov\,(p_1,\ p_2)+a_{33}cov\,(p_1,\ \omega_3), \tag{43}$$

$$cov\,(p_2,\ p_3)=a_{13}cov\,(p_1,\ p_2)+a_{23}\Sigma_{p_2}+a_{33}cov\,(p_2,\ \omega_3), \tag{44}$$

$$\Sigma_{\omega_3}=(K_I b_{k_I 13}+K_3 b_{k_3 13})\{cov\,(p_1,\ \omega_3) \tag{45}$$

$$+K_I cov\,(S_{uk_I},\ p_1)' b_{i_I u3}+K_3 cov\,(S_{uk_3},\ p_1)' b_{k_3 u3}$$

$$+K_I cov\,(S_{\theta k_I},\ p_1)' b_{k_I \theta 3}+K_3 cov\,(S_{\theta k_3},\ p_1)' b_{k_3 \theta 3}\} \tag{46}$$

$$+(K_I b_{k_I 23}+K_3 b_{k_3 23})\{cov\,(p_2,\ \omega_3)$$

$$+K_I cov\,(S_{uk_I},\ p_2)' b_{k_I u3}+K_3 cov\,(S_{uk_3},\ p_2)' b_{k_3 u3}$$

$$+K_I cov\,(S_{\theta k_I},\ p_2)' b_{k_I \theta 3}+K_3 cov\,(S_{\theta k_3},\ p_2)' b_{k_3 \theta 3}\} \tag{47}$$

$$+K_I b'_{k_I u3}(\Sigma_u+\Sigma_{\varepsilon_u})b_{k_I u3}+K_I(K_I-1)b'_{k_I u3}\Sigma_u b_{k_I u3}$$

$$+K_3 b'_{k_3 u3}(\Sigma_u+\Sigma_{\varepsilon_u})b_{k_3 u3}+K_3(K_3-1)b'_{k_3 u3}\Sigma_u b_{k_3 u3}$$

$$+2K_I K_3 b'_{k_3 u3}\Sigma_u b_{k_I u3}$$

$$+K_I b'_{k_I \theta 3}(\Sigma_\theta+\Sigma_{\varepsilon_\theta})b_{k_I \theta 3}+K_I(K_I-1)b'_{k_I \theta 3}\Sigma_\theta b_{k_I \theta 3}$$

$$+K_3 b'_{k_3 \theta 3}(\Sigma_\theta+\Sigma_{\varepsilon_\theta})b_{k_3 \theta 3}+K_3(K_3-1)b'_{k_3 \theta 3}\Sigma_\theta b_{k_3 \theta 3}$$

$$+2K_I K_3 b'_{k_3 \theta 3}\Sigma_u b_{k_I \theta 3}+\Sigma_z(3,\ 3).$$

4.2 Traders

A trader k has the following expectation at $t=1$:

$$E_1^k(v\,(1))=\bar{v}\,(1)+\frac{\Sigma_u(1,\ 1)}{\Sigma_u(1,\ 1)+\Sigma_{\varepsilon_u}(1,\ 1)}(S_{uk}(1)-\bar{u}\,(1)) \tag{48}$$

$$+\frac{\Sigma_\theta(1,\ 1)}{\Sigma_\theta(1,\ 1)+\Sigma_{\varepsilon_\theta}(1,\ 1)}(S_{\theta k}(1)-\bar{\theta}\,(1)),$$

$$E_1^k(v\,(2))=\bar{v}\,(2)+\frac{\Sigma_u(2,\ 2)}{\Sigma_u(2,\ 2)+\Sigma_{\varepsilon_u}(2,\ 2)}(S_{uk}(2)-\bar{u}\,(2)) \tag{49}$$

$$+\frac{1}{2}\frac{\Sigma_\theta(1,\ 1)}{\Sigma_\theta(1,\ 1)+\Sigma_{\varepsilon_\theta}(1,\ 1)}(S_{\theta k}(1)-\bar{\theta}\,(1)),$$

$$+\frac{1}{2}\frac{\Sigma_\theta(2,\ 2)}{\Sigma_\theta(2,\ 2)+\Sigma_{\varepsilon_\theta}(2,\ 2)}(S_{\theta k}(2)-\bar{\theta}\,(2)),$$

$$E_1^k(v\,(3))=\bar{v}\,(3)+\frac{\Sigma_u(3,\ 3)}{\Sigma_u(3,\ 3)+\Sigma_{\varepsilon_u}(3,\ 3)}(S_{uk}(3)-\bar{u}\,(3)) \tag{50}$$

$$+\frac{\Sigma_\theta(2,\ 2)}{\Sigma_\theta(2,\ 2)+\Sigma_{\varepsilon_\theta}(2,\ 2)}(S_{\theta k}(2)-\bar{\theta}\,(2)),$$

$$E_1^k(\omega_1)=\bar{\omega}_1+[cov\,(S_{uk},\ \omega_1)' \quad cov\,(S_{\theta k},\ \omega_1)'] \tag{51}$$

$$\cdot\begin{bmatrix}\Sigma_u+\Sigma_{\varepsilon_u} & 0\\ 0 & \Sigma_\theta+\Sigma_{\varepsilon_\theta}\end{bmatrix}^{-1}\begin{bmatrix}S_{uk}-\bar{u}\\ S_{\theta k}-\bar{\theta}\end{bmatrix},$$

$$E_1^k(p_1)=\bar{p}_1+[cov(S_{uk},p_1)' \quad cov(S_{\theta k},p_1)'] \cdot \begin{bmatrix} \Sigma_u+\Sigma_{\varepsilon_u} & 0 \\ 0 & \Sigma_\theta+\Sigma_{\varepsilon_\theta} \end{bmatrix}^{-1} \begin{bmatrix} S_{uk}-\bar{u} \\ S_{\theta k}-\bar{\theta} \end{bmatrix}, \tag{52}$$

$$E_1^k(\omega_2)=\bar{\omega}_2+[cov(S_{uk},\omega_2)' \quad cov(S_{\theta k},\omega_2)'] \cdot \begin{bmatrix} \Sigma_u+\Sigma_{\varepsilon_u} & 0 \\ 0 & \Sigma_\theta+\Sigma_{\varepsilon_\theta} \end{bmatrix}^{-1} \begin{bmatrix} S_{uk}-\bar{u} \\ S_{\theta k}-\bar{\theta} \end{bmatrix}, \tag{53}$$

$$E_1^k(p_2)=\bar{p}_2+[cov(S_{uk},p_2)' \quad cov(S_{\theta k},p_2)'] \cdot \begin{bmatrix} \Sigma_u+\Sigma_{\varepsilon_u} & 0 \\ 0 & \Sigma_\theta+\Sigma_{\varepsilon_\theta} \end{bmatrix}^{-1} \begin{bmatrix} S_{uk}-\bar{u} \\ S_{\theta k}-\bar{\theta} \end{bmatrix}, \tag{54}$$

$$E_1^k(p_3)=\bar{p}_3+[cov(S_{uk},p_3)' \quad cov(S_{\theta k},p_3)'] \cdot \begin{bmatrix} \Sigma_u+\Sigma_{\varepsilon_u} & 0 \\ 0 & \Sigma_\theta+\Sigma_{\varepsilon_\theta} \end{bmatrix}^{-1} \begin{bmatrix} S_{uk}-\bar{u} \\ S_{\theta k}-\bar{\theta} \end{bmatrix}. \tag{55}$$

A trader k has the following expectation at $t=2$:

$$E_2^k(v(2))=\bar{v}(2)+[cov(v(2),p_1) \quad [0 \quad \Sigma_u(2,2) \quad 0] \quad [\tfrac{1}{2}\Sigma_\theta(1,1) \quad \tfrac{1}{2}\Sigma_\theta(2,2)]] \cdot \begin{bmatrix} \Sigma_{p_1} & cov(S_{uk},p_1)' & cov(S_{\theta k},p_1)' \\ cov(S_{uk},p_1) & \Sigma_u+\Sigma_{\varepsilon_u} & 0 \\ cov(S_{\theta k},p_1) & 0 & \Sigma_\theta+\Sigma_{\varepsilon_\theta} \end{bmatrix}^{-1} \begin{bmatrix} p_1-\bar{p}_1 \\ S_{uk}-\bar{u} \\ S_{\theta k}-\bar{\theta} \end{bmatrix}, \tag{56}$$

$$E_2^k(v(3))=\bar{v}(3)+[cov(v(3),p_1) \quad [0 \quad 0 \quad \Sigma_u(3,3)] \quad [0 \quad \Sigma_\theta(2,2)]] \cdot \begin{bmatrix} \Sigma_{p_1} & cov(S_{uk},p_1)' & cov(S_{\theta k},p_1)' \\ cov(S_{uk},p_1) & \Sigma_u+\Sigma_{\varepsilon_u} & 0 \\ cov(S_{\theta k},p_1) & 0 & \Sigma_\theta+\Sigma_{\varepsilon_\theta} \end{bmatrix}^{-1} \begin{bmatrix} p_1-\bar{p}_1 \\ S_{uk}-\bar{u} \\ S_{\theta k}-\bar{\theta} \end{bmatrix}, \tag{57}$$

$$E_2^k(\omega_2)=\bar{\omega}_2+[cov(p_1,\omega_2) \quad cov(S_{uk},\omega_2)' \quad cov(S_{\theta k},\omega_2)'] \cdot \begin{bmatrix} \Sigma_{p_1} & cov(S_{uk},p_1)' & cov(S_{\theta k},p_1)' \\ cov(S_{uk},p_1) & \Sigma_u+\Sigma_{\varepsilon_u} & 0 \\ cov(S_{\theta k},p_1) & 0 & \Sigma_\theta+\Sigma_{\varepsilon_\theta} \end{bmatrix}^{-1} \begin{bmatrix} p_1-\bar{p}_1 \\ S_{uk}-\bar{u} \\ S_{\theta k}-\bar{\theta} \end{bmatrix}, \tag{58}$$

$$E_2^k(p_2)=\bar{p}_2+[cov(p_1,p_2) \quad cov(S_{uk},p_2)' \quad cov(S_{\theta k},p_2)'] \cdot \begin{bmatrix} \Sigma_{p_1} & cov(S_{uk},p_1)' & cov(S_{\theta k},p_1)' \\ cov(S_{uk},p_1) & \Sigma_u+\Sigma_{\varepsilon_u} & 0 \\ cov(S_{\theta k},p_1) & 0 & \Sigma_\theta+\Sigma_{\varepsilon_\theta} \end{bmatrix}^{-1} \begin{bmatrix} p_1-\bar{p}_1 \\ S_{uk}-\bar{u} \\ S_{\theta k}-\bar{\theta} \end{bmatrix}, \tag{59}$$

$$E_2^k(p_3)=\bar{p}_3+[cov(p_1,p_3) \quad cov(S_{uk},p_3)' \quad cov(S_{\theta k},p_3)'] \cdot \begin{bmatrix} \Sigma_{p_1} & cov(S_{uk},p_1)' & cov(S_{\theta k},p_1)' \\ cov(S_{uk},p_1) & \Sigma_u+\Sigma_{\varepsilon_u} & 0 \\ cov(S_{\theta k},p_1) & 0 & \Sigma_\theta+\Sigma_{\varepsilon_\theta} \end{bmatrix}^{-1} \begin{bmatrix} p_1-\bar{p}_1 \\ S_{uk}-\bar{u} \\ S_{\theta k}-\bar{\theta} \end{bmatrix}. \tag{60}$$

A trader k has the following expectation at $t=3$:

$$E_3^k(v(3)) = \bar{v} + [cov(v(3), p_1) \quad cov(v(3), p_2) \quad [0 \quad 0 \quad \Sigma_u(3,3)] \quad [0 \quad \Sigma_\theta(2,2)]] \tag{61}$$

$$\cdot \begin{bmatrix} \Sigma_{p_1} & cov(p_1, p_2) & cov(S_{uk}, p_1)' & cov(S_{\theta k}, p_1)' \\ cov(p_1, p_2) & \Sigma_{p_2} & cov(S_{uk}, p_2)' & cov(S_{\theta k}, p_2)' \\ cov(S_{uk}, p_1) & cov(S_{uk}, p_2) & \Sigma_u + \Sigma_{\varepsilon_u} & 0 \\ cov(S_{\theta k}, p_1) & cov(S_{\theta k}, p_2) & 0 & \Sigma_\theta + \Sigma_{\varepsilon_\theta} \end{bmatrix}^{-1} \begin{bmatrix} p_1 - \bar{p}_1 \\ p_2 - \bar{p}_2 \\ S_{uk} - \bar{u} \\ S_{\theta k} - \bar{\theta} \end{bmatrix},$$

$$E_3^k(p_3) = \bar{p}_3 + [cov(p_1, p_3) \quad cov(p_2, p_3) \quad cov(S_{uk}, p_3)' \quad cov(S_{\theta k}, p_3)'] \tag{62}$$

$$\cdot \begin{bmatrix} \Sigma_{p_1} & cov(p_1, p_2) & cov(S_{uk}, p_1)' & cov(S_{\theta k}, p_1)' \\ cov(p_1, p_2) & \Sigma_{p_2} & cov(S_{uk}, p_2)' & cov(S_{\theta k}, p_2)' \\ cov(S_{uk}, p_1) & cov(S_{uk}, p_2) & \Sigma_u + \Sigma_{\varepsilon_u} & 0 \\ cov(S_{\theta k}, p_1) & cov(S_{\theta k}, p_2) & 0 & \Sigma_\theta + \Sigma_{\varepsilon_\theta} \end{bmatrix}^{-1} \begin{bmatrix} p_1 - \bar{p}_1 \\ p_2 - \bar{p}_2 \\ S_{uk} - \bar{u} \\ S_{\theta k} - \bar{\theta} \end{bmatrix}.$$

4.3　Market Makers

A market maker has the following expectation at $t=1$:

$$E_1^M(v(1)) = \bar{v}(1) + \frac{cov(v(1), \omega_1)}{\Sigma_{\omega_1}}(\omega_1 - \bar{\omega}_1). \tag{63}$$

A market maker has the following expectation at $t=2$:

$$E_2^M(v(2)) = \bar{v}(2) + [cov(v(2), p_1) \quad cov(v(2), \omega_2)] \tag{64}$$

$$\cdot \begin{bmatrix} \Sigma_{p_1} & cov(p_1, \omega_2) \\ cov(p_1, \omega_2) & \Sigma_{\omega_2} \end{bmatrix}^{-1} \begin{bmatrix} p_1 - \bar{p}_1 \\ \omega_2 - \bar{\omega}_2 \end{bmatrix}.$$

A market maker has the following expectation at $t=3$:

$$E_3^M(v(3)) = \bar{v}(3) + [cov(v(3), p_1) \quad cov(v(3), p_2) \quad cov(v(3), \omega_3)] \tag{65}$$

$$\cdot \begin{bmatrix} \Sigma_{p_1} & cov(p_1, p_2) & cov(p_1, \omega_3) \\ cov(p_1, p_2) & \Sigma_{p_2} & cov(p_2, \omega_3) \\ cov(p_1, \omega_3) & cov(p_2, \omega_3) & \Sigma_{\omega_3} \end{bmatrix}^{-1} \begin{bmatrix} p_1 - \bar{p}_1 \\ p_2 - \bar{p}_2 \\ \omega_3 - \bar{\omega}_3 \end{bmatrix}.$$

5　Backward Induction

5.1　Period 3

A trader k maximizes

$$E_3^k[x_{k3}(v(3) - p_3)]. \tag{66}$$

The first order condition is

$$E_3^k(v(3)) - E_3^k(p_3) - a_{33}x_{k3} = 0, \tag{67}$$

and we obtain

$$x_{k3}=\frac{1}{a_{33}}[\bar{v}(3)-\bar{p}_3] \tag{68}$$

$$+\frac{1}{a_{33}}([cov(v(3),p_1) \quad cov(v(3),p_2) \quad [0 \quad 0 \quad \Sigma_u(3,3)] \quad [0 \quad \Sigma_\theta(2,2)]]$$

$$-[cov(p_1,p_3) \quad cov(p_2,p_3) \quad cov(S_{uk},p_3)' \quad cov(S_{\theta k},p_3)'])$$

$$\cdot\begin{bmatrix} \Sigma_{p_1} & cov(p_1,p_2) & cov(S_{uk},p_1)' & cov(S_{\theta k},p_1)' \\ cov(p_1,p_2) & \Sigma_{p_2} & cov(S_{uk},p_2)' & cov(S_{\theta k},p_2)' \\ cov(S_{uk},p_1) & cov(S_{uk},p_2) & \Sigma_u+\Sigma_{\varepsilon_u} & 0 \\ cov(S_{\theta k},p_1) & cov(S_{\theta k},p_2) & 0 & \Sigma_\theta+\Sigma_{\varepsilon_\theta} \end{bmatrix}^{-1}\cdot\begin{bmatrix} p_1-\bar{p}_1 \\ p_2-\bar{p}_2 \\ S_{uk}-\bar{u} \\ S_{\theta K}-\bar{\theta} \end{bmatrix}.$$

By comparison with

$$x_{k3}(p_1,p_2,S_{uk},S_{\theta k})=b_{k03}+b_{k13}(p_1-\bar{p}_1)+b_{k23}(p_2-\bar{p}_2)+b'_{ku3}(S_{uk}-\bar{u})+b'_{k\theta 3}(S_{\theta k}-\bar{\theta}), \tag{69}$$

we obtain

$$b_{k03}=\frac{1}{a_{33}}[\bar{v}(3)-\bar{p}_3], \tag{70}$$

$$[b_{k13} \quad b_{k23} \quad b'_{ku3} \quad b'_{k\theta 3}] \tag{71}$$

$$=\frac{1}{a_{33}}([cov(v(3),p_1) \quad cov(v(3),p_2) \quad [0 \quad 0 \quad \Sigma_u(3,3)] \quad [0 \quad \Sigma_\theta(2,2)]]$$

$$-[cov(p_1,p_3) \quad cov(p_2,p_3) \quad cov(S_{uk},p_3)' \quad cov(S_{\theta k},p_3)'])$$

$$\cdot\begin{bmatrix} \Sigma_{p_1} & cov(p_1,p_2) & cov(S_{uk},p_1)' & cov(S_{\theta k},p_1)' \\ cov(p_1,p_2) & \Sigma_{p_2} & cov(S_{uk},p_2)' & cov(S_{\theta k},p_2)' \\ cov(S_{uk},p_1) & cov(S_{uk},p_2) & \Sigma_u+\Sigma_{\varepsilon_u} & 0 \\ cov(S_{\theta k},p_1) & cov(S_{\theta k},p_2) & 0 & \Sigma_\theta+\Sigma_{\varepsilon_\theta} \end{bmatrix}^{-1}.$$

Since market makers are perfectly competitive,

$$p_3=E_3^M(v(3))=\bar{v}(3)+[cov(v(3),p_1) \quad cov(v(3),p_2) \quad cov(v(3),\omega_3)] \tag{72}$$

$$\cdot\begin{bmatrix} \Sigma_{p_1} & cov(p_1,p_2) & cov(p_1,\omega_3) \\ cov(p_1,p_2) & \Sigma_{p_2} & cov(p_2,\omega_3) \\ cov(p_1,\omega_3) & cov(p_2,\omega_3) & \Sigma_{\omega_3} \end{bmatrix}^{-1}\begin{bmatrix} p_1-\bar{p}_1 \\ p_2-\bar{p}_2 \\ \omega_3-\bar{\omega}_3 \end{bmatrix}.$$

By comparison with

$$p_3(p_1,p_2,\omega_3)=a_{03}+a_{13}(p_1-\bar{p}_1)+a_{23}(p_2-\bar{p}_2)+a_{33}(\omega_3-\bar{\omega}_3), \tag{73}$$

we obtain

$$a_{03}=\bar{v}(3), \tag{74}$$

$$[a_{13} \quad a_{23} \quad a_{33}]=[cov(v(3),p_1) \quad cov(v(3),p_2) \quad cov(v(3),\omega_3)] \tag{75}$$

$$\cdot\begin{bmatrix} \Sigma_{p_1} & cov(p_1,p_2) & cov(p_1,\omega_3) \\ cov(p_1,p_2) & \Sigma_{p_2} & cov(p_2,\omega_3) \\ cov(p_1,\omega_3) & cov(p_2,\omega_3) & \Sigma_{\omega_3} \end{bmatrix}^{-1}.$$

Therefore

$$p_3 = a_{03} + \alpha_{13}(p_1 - \bar{p}_1) + \alpha_{23}(\omega_2 - \bar{\omega}_2) \tag{76}$$
$$+ a_{33}\left(\sum_k b'_{ku3} S_{uk} + \sum_k b'_{k\theta 3} S_{\theta k} + z(3) - \bar{z}(3)\right)$$

$$x_{k_I 3} = b_{k_I 03} + \beta_{13}(p_1 - \bar{p}_1) + \beta_{23}(\omega_2 - \bar{\omega}_2) \tag{77}$$
$$+ b'_{k_I u3}(S_{uk_I} - \bar{u}) + b'_{k_I \theta 3}(S_{\theta k_I} - \bar{\theta}),$$

where

$$\alpha_{13} = a_{13} + a_{33}(K_I b_{k_I 13} + K_3 b_{k_3 13}) \tag{78}$$
$$+ a_{12}\{a_{23} + a_{33}(K_I b_{k_I 23} + K_3 b_{k_3 23})\},$$

$$\alpha_{23} = a_{22}\{a_{23} + a_{33}(K_I b_{k_I 23} + K_3 b_{k_3 23})\}, \tag{79}$$

$$\beta_{13} = b_{k_I 13} + a_{12} b_{k_I 23}, \tag{80}$$

$$\beta_{23} = a_{22} b_{k_I 23}. \tag{81}$$

5.2 Period 2

A domestic trader k_2 maximizes

$$E_2^{k_2}[x_{k_2 2}(v(2) - p_2)]. \tag{82}$$

The first order condition is

$$E_2^{k_2}(v(2)) E_2^{k_2}(p_2) - a_{22} x_{k_2 2} = 0, \tag{83}$$

and we obtain

$$x_{k_2 2} = \frac{1}{a_{22}}[\bar{v}(2) - \bar{p}_2] \tag{84}$$
$$+ \frac{1}{a_{22}}\{([cov(v(2), p_1) \quad [0 \quad \Sigma_u(2,2) \quad 0] \quad [\tfrac{1}{2}\Sigma_\theta(1,1) \quad \tfrac{1}{2}\Sigma_\theta(2,2)]]$$
$$- cov(p_1, p_2) \quad cov(S_{uk_2}, p_2)' \quad cov(S_{\theta k_2}, p_2)'])$$
$$\cdot \begin{bmatrix} \Sigma_{p_1} & cov(S_{uk_2}, p_1)' & cov(S_{\theta k_2}, p_1)' \\ cov(S_{uk_2}, p_1) & \Sigma_u + \Sigma_{\varepsilon_u} & 0 \\ cov(S_{\theta k_2}, p_1) & 0 & \Sigma_\theta + \Sigma_{\varepsilon_\theta} \end{bmatrix}^{-1} \} \begin{bmatrix} p_1 - \bar{p}_1 \\ S_{uk_2} - \bar{u} \\ S_{\theta k_2} - \bar{\theta} \end{bmatrix}.$$

By comparison with

$$x_{k_2 2}(p_1, S_{uk_2}, S_{\theta k_2}) = b_{k_2 02} + b_{k_2 12}(p_1 - \bar{p}_1) + b'_{k_2 u2}(S_{uk_2} - \bar{u}) + b'_{k_2 \theta 2}(S_{\theta k_2} - \bar{\theta}), \tag{85}$$

we obtain

$$b_{k_2 02} = \frac{1}{a_{22}}\bar{v}(2) - \frac{1}{a_{22}}\bar{p}_2, \tag{86}$$

$$[b_{k_2 12} \quad b'_{k_2 u2} \quad b'_{k_2 \theta 2}] \tag{87}$$

$$= \frac{1}{a_{22}}([cov(v(2), p_1) \quad [0 \quad \Sigma_u(2,2) \quad 0] \quad [\tfrac{1}{2}\Sigma_\theta(1,1) \quad \tfrac{1}{2}\Sigma_\theta(2,2)]] \tag{88}$$
$$- [cov(p_1, p_2) \quad cov(S_{uk_2}, p_2)' \quad cov(S_{\theta k_2}, p_2)'])$$

$$\cdot\begin{bmatrix} \Sigma_{p_1} & cov(S_{uk_2}, p_1)' & cov(S_{\theta k_2}, p_1)' \\ cov(S_{uk_2}, p_1) & \Sigma_u + \Sigma_{\varepsilon u} & 0 \\ cov(S_{\theta k_2}, p_1) & 0 & \Sigma_\theta + \Sigma_{\varepsilon\theta} \end{bmatrix}^{-1}.$$

An international trader k_I maximizes

$$E_2^{k_I}[x_{k_I 2}(v(2)-p_2)+x_{k_I 3}(v(3)-p_3)]. \tag{89}$$

The first order condition is

$$E_2^{k_I}(v(2))-E_2^{k_I}(p_2)-a_{22}x_{k_I 2}+\beta_{23}\{E_2^{k_I}(v(3))-E_2^{k_I}(p_3)\}-\alpha_{23}E_2^{k_I}(x_{k_I 3})=0, \tag{90}$$

and we obtain

$$\begin{aligned} x_{k_I 2} &= \frac{1}{a_{22}}[\bar{v}(2)-\bar{p}_2+\beta_{23}\{\bar{v}(3)-\bar{p}_3\}-\alpha_{23}\beta_{03}] \\ &+\frac{1}{a_{22}}\{-\alpha_{23}[\beta_{13} \quad b'_{k_I u3} \quad b'_{k_I \theta 3}] \\ &+([cov(v(2), p_1) \quad [0 \quad \Sigma_u(2, 2) \quad 0] \quad [\tfrac{1}{2}\Sigma_\theta(1, 1) \quad \tfrac{1}{2}\Sigma_\theta(2, 2)]] \\ &-[cov(p_1, p_2) \quad cov(S_{uk_I}, p_2)' \quad cov(S_{\theta k_I}, p_2)'] \\ &+\beta_{23}[cov(v(3), p_1) \quad [0 \quad 0 \quad \Sigma_u(3, 3)] \quad [0 \quad \Sigma_\theta(2, 2)]] \\ &-\beta_{23}[cov(p_1, p_3) \quad cov(S_{uk_I}, p_3)' \quad cov(S_{\theta k_I}, p_3)'] \\ &-\alpha_{23}\beta_{23}[cov(p_1, \omega_2) \quad cov(S_{uk_I}, \omega_2)' \quad cov(S_{\theta k_I}, \omega_2)']) \\ &\cdot\begin{bmatrix} \Sigma_{p_1} & cov(S_{uk_I}, p_1)' & cov(S_{\theta k_I}, p_1)' \\ cov(S_{uk_I}, p_1) & \Sigma_u + \Sigma_{\varepsilon u} & 0 \\ cov(S_{\theta k_I}, p_1) & 0 & \Sigma_\theta + \Sigma_{\varepsilon\theta} \end{bmatrix}^{-1}\} \begin{bmatrix} p_1-\bar{p}_1 \\ S_{uk_I}-\bar{u} \\ S_{\theta k_I}-\bar{\theta} \end{bmatrix}. \end{aligned} \tag{91}$$

By comparison with

$$x_{k_I 2}(p_1, S_{uk_I}, S_{\theta k_I})=b_{k_I 02}+b_{k_I 12}(p_1-\bar{p}_1)+b'_{k_I u2}(S_{uk_I}-\bar{u})+b'_{k_I \theta 2}(S_{\theta k_I}-\bar{\theta}), \tag{92}$$

we obtain

$$b_{k_I 02}=\frac{1}{a_{22}}[\bar{v}(2)-\bar{p}_2+\beta_{23}\{\bar{v}(3)-\bar{p}_3\}-\alpha_{23}\beta_{03}], \tag{93}$$

$$\begin{aligned} &[b_{k_I 12} \quad b'_{k_I u2} \quad b'_{k_I \theta 2}] \\ &=\frac{1}{a_{22}}\{-\alpha_{23}[\beta_{13} \quad b'_{k_I u3} \quad b'_{k_I \theta 3}] \\ &+([cov(v(2), p_1) \quad [0 \quad \Sigma_u(2, 2) \quad 0] \quad [\tfrac{1}{2}\Sigma_\theta(1, 1) \quad \tfrac{1}{2}\Sigma_\theta(2, 2)]] \\ &-[cov(p_1, p_2) \quad cov(S_{uk_I}, p_2)' \quad cov(S_{\theta k_I}, p_2)'] \\ &+\beta_{23}[cov(v(3), p_1) \quad [0 \quad 0 \quad \Sigma_u(3, 3)] \quad [0 \quad \Sigma_\theta(2, 2)]] \\ &-\beta_{23}[cov(p_1, p_3) \quad cov(S_{uk_I}, p_3)' \quad cov(S_{\theta k_I}, p_3)'] \\ &-\alpha_{23}\beta_{23}[cov(p_1, \omega_2) \quad cov(S_{uk_I}, \omega_2)' \quad cov(S_{\theta k_I}, \omega_2)']) \\ &\cdot\begin{bmatrix} \Sigma_{p_1} & cov(S_{uk_I}, p_1)' & cov(S_{\theta k_I}, p_1)' \\ cov(S_{uk_I}, p_1) & \Sigma_u + \Sigma_{\varepsilon u} & 0 \\ cov(S_{\theta k_I}, p_1) & 0 & \Sigma_\theta + \Sigma_{\varepsilon\theta} \end{bmatrix}^{-1}\}. \end{aligned} \tag{94}$$

Since market makers are perfectly competitive,

$$p_2=E_2^M(v(2))=\bar{v}(2)+[cov(v(2),\ p_1)\quad cov(v(2),\ \omega_2)] \cdot \begin{bmatrix} \Sigma_{p_1} & cov(p_1,\ \omega_2) \\ cov(p_1,\ \omega_2) & \Sigma_{\omega_2} \end{bmatrix}^{-1} \begin{bmatrix} p_1-\bar{p}_1 \\ \omega_2-\bar{\omega}_2 \end{bmatrix}. \tag{95}$$

By comparison with

$$p_2(p_1,\ \omega_2)=a_{02}+a_{12}(p_1-\bar{p}_1)+a_{22}(\omega_2-\bar{\omega}_2), \tag{96}$$

we obtain

$$a_{02}=\bar{v}(2), \tag{97}$$

$$[a_{12}\quad a_{22}]=[cov(v(2),\ p_1)\quad cov(v(2),\ \omega_2)] \cdot \begin{bmatrix} \Sigma_{p_1} & cov(p_1,\ \omega_2) \\ cov(p_1,\ \omega_2) & \Sigma_{\omega_2} \end{bmatrix}^{-1}. \tag{98}$$

Therefore,

$$p_2=a_{02}+\gamma_{12}(\omega_1-\bar{\omega}_1) +a_{22}\Big(\sum_k b'_{ku2}S_{uk}+\sum_k b'_{k02}S_{\theta k}+z(2)-\bar{z}(2)\Big), \tag{99}$$

$$x_{k_I2}=b_{k_I02}+\delta_{12}(\omega_1-\bar{\omega}_1) +b'_{k_Iu2}(S_{uk_I}-\bar{u})+b'_{k_I\theta2}(S_{\theta k_I}-\bar{\theta}), \tag{100}$$

$$p_3=a_{03}+\alpha_{12}(\omega_1-\bar{\omega}_1) +\alpha_{23}\Big(\sum_k b'_{ku2}S_{uk}+\sum_k b'_{k\theta2}S_{\theta k}+z(2)-\bar{z}(2)\Big) +a_{33}\Big(\sum_k b'_{ku3}S_{uk}+\sum_k b'_{k\theta3}S_{\theta k}+z(3)-\bar{z}(3)\Big), \tag{101}$$

$$x_{k_I3}=b_{k_I03}+\beta_{12}(\omega_1-\bar{\omega}_1) +\beta_{23}\Big(\sum_k b'_{ku2}S_{uk}+\sum_k b'_{k\theta2}S_{\theta k}+z(2)-\bar{z}(2)\Big) +b'_{k_Iu3}(S_{uk_I}-\bar{u})+b'_{k_I\theta3}(S_{\theta k_I}-\bar{\theta}), \tag{102}$$

where

$$\gamma_{12}=a_{11}\{a_{12}+a_{22}(K_Ib_{k_I12}+K_2b_{k_212})\}, \tag{103}$$

$$\delta_{12}=a_{11}b_{k_I12}. \tag{104}$$

$$\alpha_{12}=a_{11}\{\alpha_{13}+\alpha_{23}(K_Ib_{k_I12}+K_2b_{k_212})\}, \tag{105}$$

$$\beta_{12}=a_{11}\{\beta_{13}+\beta_{23}(K_Ib_{k_I12}+K_2b_{k_212})\}. \tag{106}$$

5.3 Period 1

A domestic trader k_1 maximizes

$$E_1^{k_1}[x_{k_1 1}(v(1)-p_1)]. \tag{107}$$

The first order condition is

$$E_1^{k_1}(v(1))-E_1^{k_1}(p_1)-a_{11}x_{k_1 1}=0, \tag{108}$$

and we obtain

$$\begin{aligned} x_{k_1 1}=&\frac{1}{a_{11}}[\bar{v}(1)-\bar{p}_1] \\ &+\frac{1}{a_{11}}([[\Sigma_u(1,1) \quad 0 \quad 0] \quad [\Sigma_\theta(1,1) \quad 0]] \\ &-[cov(S_{uk_1},p_1)' \quad cov(S_{\theta k_1},p_1)']) \\ &\cdot\begin{bmatrix}\Sigma_u+\Sigma_{\varepsilon_u} & 0 \\ 0 & \Sigma_\theta+\Sigma_{\varepsilon_\theta}\end{bmatrix}^{-1}\begin{bmatrix}S_{uk_1}-\bar{u} \\ S_{\theta k_1}-\bar{\theta}\end{bmatrix}. \end{aligned} \tag{109}$$

By comparison with

$$x_{k_1 1}(S_{uk_1},S_{\theta k_1})=b_{k_1 01}+b'_{k_1 u1}(S_{uk_1}-\bar{u})+b'_{k_1\theta 1}(S_{\theta k_1}-\bar{\theta}), \tag{110}$$

we obtain

$$b_{k_1 01}=\frac{1}{a_{11}}[\bar{v}(1)-\bar{p}_1], \tag{111}$$

$$\begin{aligned} &[b'_{k_1 u1} \quad b'_{k_1\theta 1}] \\ &=\frac{1}{a_{11}}([[\Sigma_u(1,1) \quad 0 \quad 0] \quad [\Sigma_\theta(1,1) \quad 0]] \\ &\quad -[cov(S_{uk_1},p_1)' \quad cov(S_{\theta k_1},p_1)']) \\ &\quad \cdot\begin{bmatrix}\Sigma_u+\Sigma_{\varepsilon_u} & 0 \\ 0 & \Sigma_\theta+\Sigma_{\varepsilon_\theta}\end{bmatrix}^{-1}. \end{aligned} \tag{112}$$

An international trader k_I maximizes

$$E_1^{k_I}[x_{k_I 1}(v(1)-p_1)+x_{k_I 2}(v(2)-p_2)+x_{k_I 3}(v(3)-p_3)]. \tag{113}$$

The first order condition is

$$\begin{aligned} &E_1^{k_I}(v(1))-E_1^{k_I}(p_1)-a_{11}x_{k_I 1} \\ &\quad+\delta_{12}\{E_1^{k_I}(v(2))-E_1^{k_I}(p_2)\}-\gamma_{12}x_{k_I 2} \\ &\quad+\beta_{12}\{E_1^{k_I}(v(3))-E_1^{k_I}(p_3)\}-\alpha_{12}x_{k_I 3} \\ &\quad=0, \end{aligned} \tag{114}$$

and we obtain

$$\begin{aligned} x_{k_I 1}=&\frac{1}{a_{11}}[\bar{v}(1)-\bar{p}_1+\delta_{12}\{\bar{v}(2)-\bar{p}_2\}-\gamma_{12}\delta_{02}+\beta_{12}\{\bar{v}(3)-\bar{p}_3\}-\alpha_{12}\beta_{03}] \\ &+\frac{1}{a_{11}}\{-\gamma_{12}[b'_{k_I u3} \quad b'_{k_I\theta 3}]-\alpha_{12}[b'_{k_I u3} \quad b'_{k_I\theta 3}] \\ &+([[\Sigma_u(1,1) \quad 0 \quad 0] \quad [\Sigma_\theta(1,1) \quad 0]] \\ &-[cov(S_{uk_I},p_1)' \quad cov(S_{\theta k_I},p_1)'] \end{aligned} \tag{115}$$

$$+\delta_{12}[[0 \quad \Sigma_u(2,2) \quad 0] \quad [\tfrac{1}{2}\Sigma_\theta(1,1) \quad \tfrac{1}{2}\Sigma_\theta(2,2)]]$$
$$-\delta_{12}[cov(S_{uk_I}, p_2)' \quad cov(S_{\theta k_I}, p_2)']$$
$$-\gamma_{12}\delta_{12}[cov(S_{uk_I}, \omega_1)' \quad cov(S_{\theta k_I}, \omega_1)']$$
$$+\beta_{12}[[0 \quad 0 \quad \Sigma_u(3,3)] \quad [0 \quad \Sigma_\theta(2,2)]]$$
$$-\beta_{12}[cov(S_{uk_I}, p_3)' \quad cov(S_{\theta k_I}, p_3)']$$
$$-\alpha_{12}\beta_{13}[cov(S_{uk_I}, p_1)' \quad cov(S_{\theta k_I}, p_1)']$$
$$-\alpha_{12}\beta_{23}[cov(S_{uk_I}, \omega_2)' \quad cov(S_{\theta k_I}, \omega_2)'])$$
$$\cdot\begin{bmatrix}\Sigma_u+\Sigma_{\varepsilon_u} & 0\\ 0 & \Sigma_\theta+\Sigma_{\varepsilon_\theta}\end{bmatrix}^{-1}\Bigg\}\begin{bmatrix}S_{uk_I}-\bar{u}\\ S_{\theta k_I}-\bar{\theta}\end{bmatrix}.$$

By comparison with

$$x_{k_I1}(S_{uk_I}, S_{\theta k_I})=b_{k_I01}+b'_{k_Iu1}(S_{uk_I}-\bar{u})+b'_{k_I\theta1}(S_{\theta k_I}-\bar{\theta}), \tag{116}$$

we obtain

$$b_{k_I01}=\frac{1}{a_{11}}[\bar{v}(1)-\bar{p}_1+\delta_{12}\{\bar{v}(2)-\bar{p}_2\}-\gamma_{12}\delta_{02}+\beta_{12}\{\bar{v}(3)-\bar{p}_3\}-\alpha_{12}\beta_{03}], \tag{117}$$

$$[b'_{k_Iu1} \quad b'_{k_I\theta1}] \tag{118}$$
$$=-\frac{\gamma_{12}}{a_{11}}[b'_{k_Iu3} \quad b'_{k_I\theta3}]-\frac{\alpha_{12}}{a_{11}}[b'_{k_Iu3} \quad b'_{k_I\theta3}]$$
$$+\frac{1}{a_{11}}([[\Sigma_u(1,1) \quad 0 \quad 0] \quad [\Sigma_\theta(1,1) \quad 0]]$$
$$-[cov(S_{uk_I}, p_1)' \quad cov(S_{\theta k_I}, p_1)']$$
$$+\delta_{12}[[0 \quad \Sigma_u(2,2) \quad 0] \quad [\tfrac{1}{2}\Sigma_\theta(1,1) \quad \tfrac{1}{2}\Sigma_\theta(2,2)]]$$
$$-\delta_{12}[cov(S_{uk_I}, p_2)' \quad cov(S_{\theta k_I}, p_2)']$$
$$-\gamma_{12}\delta_{12}[cov(S_{uk_I}, \omega_1)' \quad cov(S_{\theta k_I}, \omega_1)']$$
$$+\beta_{12}[[0 \quad 0 \quad \Sigma_u(3,3)] \quad [0 \quad \Sigma_\theta(2,2)]]$$
$$-\beta_{12}[cov(S_{uk_I}, p_3)' \quad cov(S_{\theta k_I}, p_3)']$$
$$-\alpha_{12}\beta_{13}[cov(S_{uk_I}, p_1)' \quad cov(S_{\theta k_I}, p_1)']$$
$$-\alpha_{12}\beta_{23}[cov(S_{uk_I}, \omega_2)' \quad cov(S_{\theta k_I}, \omega_2)'])$$
$$\cdot\begin{bmatrix}\Sigma_u+\Sigma_{\varepsilon_u} & 0\\ 0 & \Sigma_\theta+\Sigma_{\varepsilon_\theta}\end{bmatrix}^{-1}.$$

Since market makers are perfectly competitive,

$$p_1=E_1^M(v(1))=\bar{v}(1)+\frac{cov(v(1), \omega_1)}{\Sigma_{\omega_1}}(\omega_1-\bar{\omega}_1). \tag{119}$$

By comparison with

$$p_1(\omega_1)=a_{01}+a_{11}(\omega_1-\bar{\omega}_1), \tag{120}$$

we obtain

$$a_{01} = \bar{v}(1), \tag{121}$$

$$a_{11} = \frac{cov(v(1), \omega_1)}{\Sigma_{\omega_1}}. \tag{122}$$

6 Numerical Calculations

Now the number of equations equals the number of variables to be solved. However, it is difficult to study the equilibrium prices and volume analytically. We see the result of the numerical calculations in the following. We now examine in more detail the behavior of the parameters. We focus mainly on the case where

$$K_I = 15,\ K_1 = K_2 = K_3 = 5,$$

$$\Sigma_u = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\Sigma_\theta = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\Sigma_{\varepsilon u} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix},$$

$$\Sigma_{\varepsilon \theta} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix},$$

$$\Sigma_z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

While specific patterns may be vary with the parameter values chosen, the qualitative features of those patterns are robust.

The result is

$$a_{11} = 0.4918408,$$

$$a_{12} = 0.3887709,$$

$$a_{22} = 0.2408513,$$

$$a_{13} = -0.3195859,$$

$$a_{23} = 0.823116,$$

$$a_{33} = 0.3641825,$$

$$b_{k_I u1} = (0.0817214431,\ -0.0020110471,\ 0.0007345236)',$$

$$b_{k_1 u1} = (0.080735645,\ 0.001371168,\ -0.001101785)',$$

$$b_{k_I \theta 1} = (0.0955175016,\ -0.0007001674)',$$

$b_{k_1\theta 1}=(0.097364687, 0.001615771)'$,

$b_{k_I 12}=-0.07385632$,

$b_{k_2 12}=-0.07149181$,

$b_{k_I u2}=(-0.003214054, 0.112825774, -0.003773137)'$,

$b_{k_2 u2}=(-0.001378775, 0.111838170, 0.005638745)'$,

$b_{k_I \theta 2}=(0.09559871, 0.09346503)'$,

$b_{k_2 \theta 2}=(0.09226272, 0.10370560)'$,

$b_{k_I 13}=0.04138529$,

$b_{k_3 13}=0.04122749$,

$b_{k_I 23}=-0.09773576$

$b_{k_3 23}=-0.1003121$

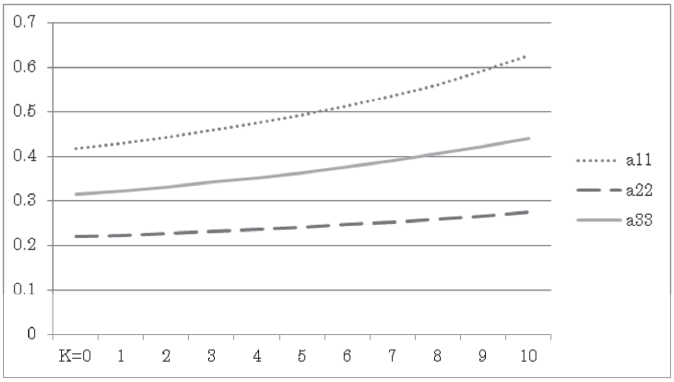
$b_{k_I u3}=(0.0002077511, -0.0043496286, 0.1099177368)'$,

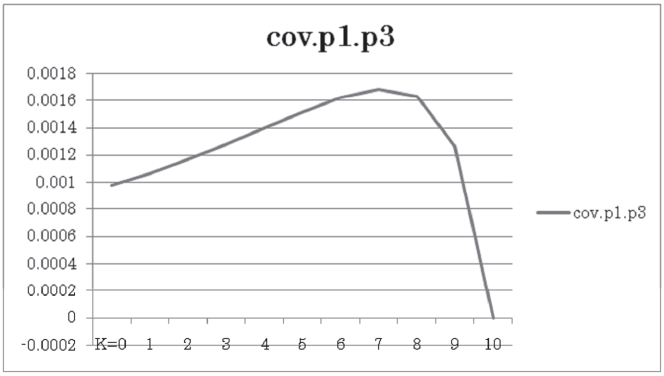
$b_{k_3 u3}=(0.0001444080, -0.0017749524, 0.1098431954)'$,

$b_{k_I \theta 3}=(-0.004478454, 0.123117411)'$,

$b_{k_3 \theta 3}=(-0.002955102, 0.124642815)'$.

In the following, we characterize an effect of the internationalization of investing on financial contagion. We consider the case where $K_1=K_2=K_3=K$, and $K_I=30-3K$ for $K=0, 1, \cdots, 10$. We are right in thinking that internationalization increases as K decreases. The results are as follows:





As internationalization increases,

1. the market impacts decrease,
2. the ex-ante variance of price of each country increases, and
3. the ex-ante covariance of prices between related countries increases.

When there is no international trader, the price of first country is not associated with the one of third country.

7 Concluding Comments

We have seen a financial contagion model with time differences. In our case, an idiosyncratic shock to other countries has complicated market impacts. Understanding the channels through which liquidity is impacted is for future research.

Note

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Reference

Caballe, Jordi & Krishnan, Murugappa, 1994. "Imperfect Competition in a Multi-security Market with Risk Neutrality," Econometrica, Econometric Society, vol. 62(3), pages 695–704, May.