愛知淑徳大学論集―人間情報学部篇　第8号　2018年3月，pp. 1–8

原著

Visualizations of partial and part correlation coefficients

偏相関係数および部分相関係数の視覚化

Shigeaki AMANO*

天　野　成　昭

Abstract

Partial and part correlations are often used in a psychological research. They represent the relationships between variables x and y after removing the effects of another variable z on x and/or y. In order to achieve a better understanding of partial and part correlations, their coefficients were visualized using surface and contour maps in this study. It was revealed that a partial or part correlation coefficient is mapped onto a curved surface like a horse saddle in a coordinate space with axes of a correlation coefficient between x and z, a correlation coefficient between y and z, and a partial or part correlation coefficient.

Keywords: partial correlation, part correlation, surface map, contour map

1. Introduction

Pearson's product-moment correlation coefficient is an index for representing the relationships between two variables x and y, and is widely employed in statistical analyses. However, it does not reflect the true relationship between two variables when a third variable z has an effect on x and/or y. That is, x and y may have a spurious correlation caused by z. In such cases, a partial or part correlation is a valid index for representing the relationships, because it removes the effects that z has on x and/or y.

A partial correlation coefficient is defined by Equation 1. It represents the relationship between x and y after removing the influence of z from both x and y.

$$r_{(x|y)(y|z)}=\frac{r_{xy}-r_{xz}r_{yz}}{\sqrt{1-r_{xz}{}^2}\sqrt{1-r_{yz}{}^2}} \tag{1}$$

where $r(x|z)(y|z)$ is a partial correlation coefficient, and r_{xy}, $r_{xz,}$ and r_{yz} are the correlation coefficients between x and y, x and z, and y and z, respectively. A partial correlation is frequently applied in psychological research (e.g., Ebisu & Iwanaga, 2016; Kashiwagi & Hirayama, 2003; Sasaki, Sugawara, & Tanno, 2005; Segerstrom, Taylor, Kemeny, & Fahey, 1998; Shibata, 2016; Shimotsukasa & Oshio, 2016; Ueyama & Sugimura, 2015; Yoshida & Murayama, 2013; Walberg & Tsai, 1983).

* 愛知淑徳大学人間情報学部

A part correlation coefficient is defined by Equation 2, which represents the relationship between x and y after removing the influence of z from x.

$$r_{(x|z)y}=\frac{r_{xy}-r_{xz}r_{yz}}{\sqrt{1-r_{xz}^{2}}} \tag{2}$$

where $r(x|z)y$ is a part correlation coefficient, and r_{xy}, r_{xz}, and r_{yz} are the correlation coefficients between x and y, x and z, and y and z, respectively. A part correlation is also called as a semi-partial correlation. It is often used to indicate a strength of relationship between a dependent variable and an independent variable removing effects of other variables in a multiple regression analysis in psychological researches (e.g., Davoudi & Chavosh, 2016; Fichten et al., 2016; Gilden & Wilson, 1995; Kajabadi, HajiMohammadi, & Pahlavani, 2016; Thorsteinsson, Brown, & Richards, 2014).

Although partial and part correlations play an important role in psychological research, it is somewhat difficult to achieve an intuitive understanding of how the correlations change as functions of r_{xy}, r_{xz}, and r_{yz}, in Equations 1 and 2. This study attempts to overcome this difficulty and convey a better understanding by visualizing partial and part correlation coefficients using surface and contour maps.

2. Map of Partial Correlation

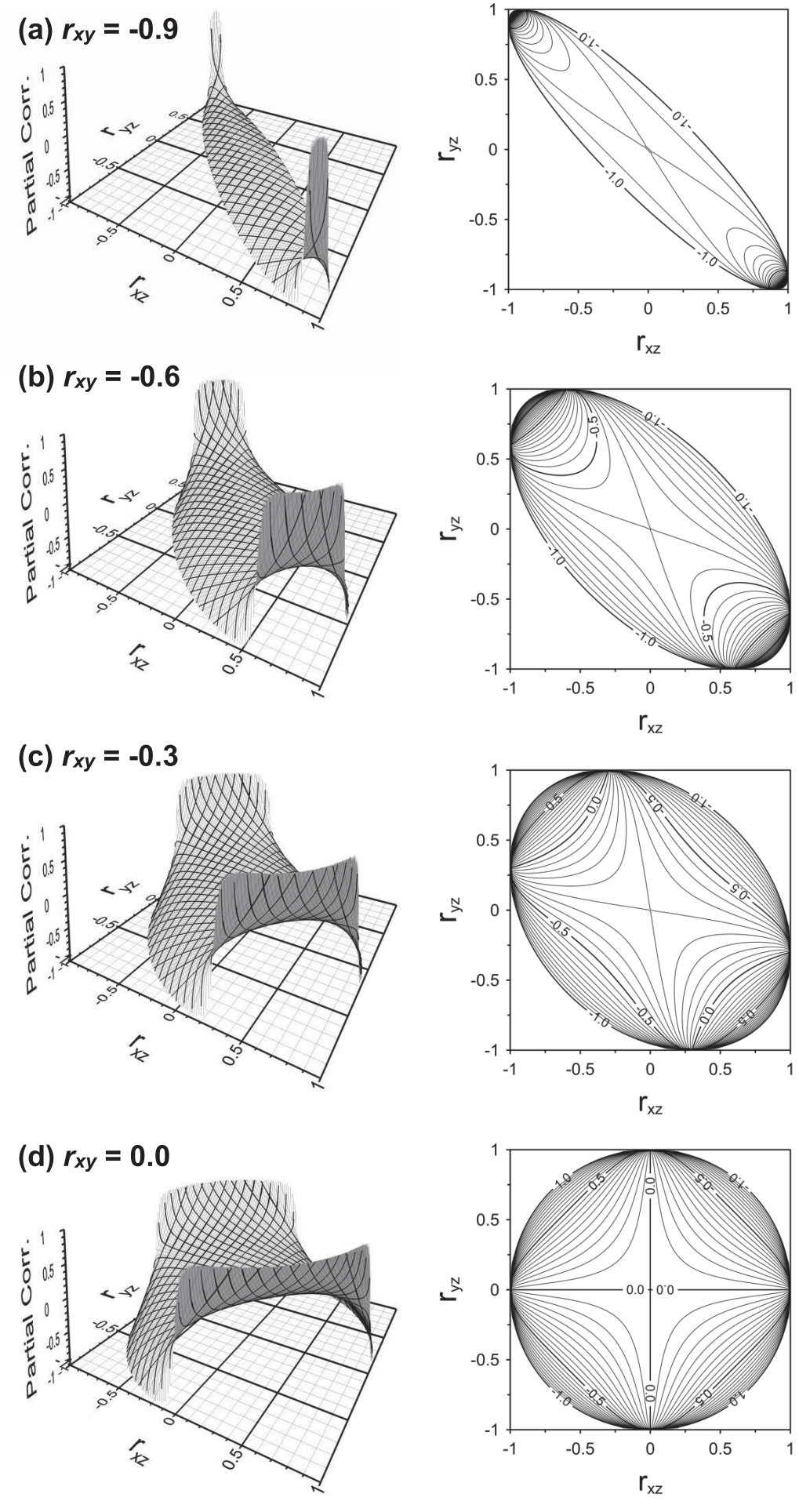
Surface and contour maps for the partial correlation coefficient defined by Equation 1 are plotted with a range of $-1.0 \leqq r_{xz} \leqq 1.0$ and $-1.0 \leqq r_{yz} \leqq 1.0$, for r_{xy} at -0.9, -0.6, -0.3, 0.0, 0.3, 0.6, and 0.9 (Figure 1). Each panel of Figure 1 indicates that a partial correlation coefficient is mapped onto a curved surface, similar to a horse saddle. The curved surface is symmetric with respect to the planes $r_{xz}=r_{yz}$ and $r_{xz}=-r_{yz}$. A shape of the curved surface changes according to r_{xy}. It should be noted that the partial correlation coefficient is equal to r_{xy} at the point of origin, $(r_{xz}, r_{yz})=(0.0, 0.0)$. This means that the height of the center of the surface represents correlation between x and y.

3. Map of Part Correlation

Surface and contour maps for the part correlation coefficient defined by Equation 2 are plotted with a range of $-1.0 \leqq r_{xz} \leqq 1.0$ and $-1.0 \leqq r_{yz} \leqq 1.0$, for r_{xy} at -0.9, -0.6, -0.3, 0.0, 0.3, 0.6, and 0.9 (Figure 2). Each panel of Figure 2 indicates that the part correlation coefficient is mapped onto a curved surface, similar to a distorted horse saddle. Unlike in Figure 1, the curved surface in Figure 2 is symmetric with respect to the vertical line through the point of origin, $(r_{xz}, r_{yz}) = (0.0, 0.0)$. It should be noted that just like the partial correlation coefficient, the part correlation coefficient is equal to r_{xy} at the origin point, and the curved shape systematically changes according to r_{xy}.

4. Discussion

This study has presented surface and contour maps of partial and part correlation coefficients. Equation 1 and 2 indicate that the partial and part correlations coefficients have similar values when r_{yz} goes to zero, but they have different values when r_{yz} goes to 1 or -1. This tendency can be seen in right panels of


(a) r_{xy} = -0.9
(b) r_{xy} = -0.6
(c) r_{xy} = -0.3
(d) r_{xy} = 0.0
Partial Corr.
r_{yz}
r_{xz}

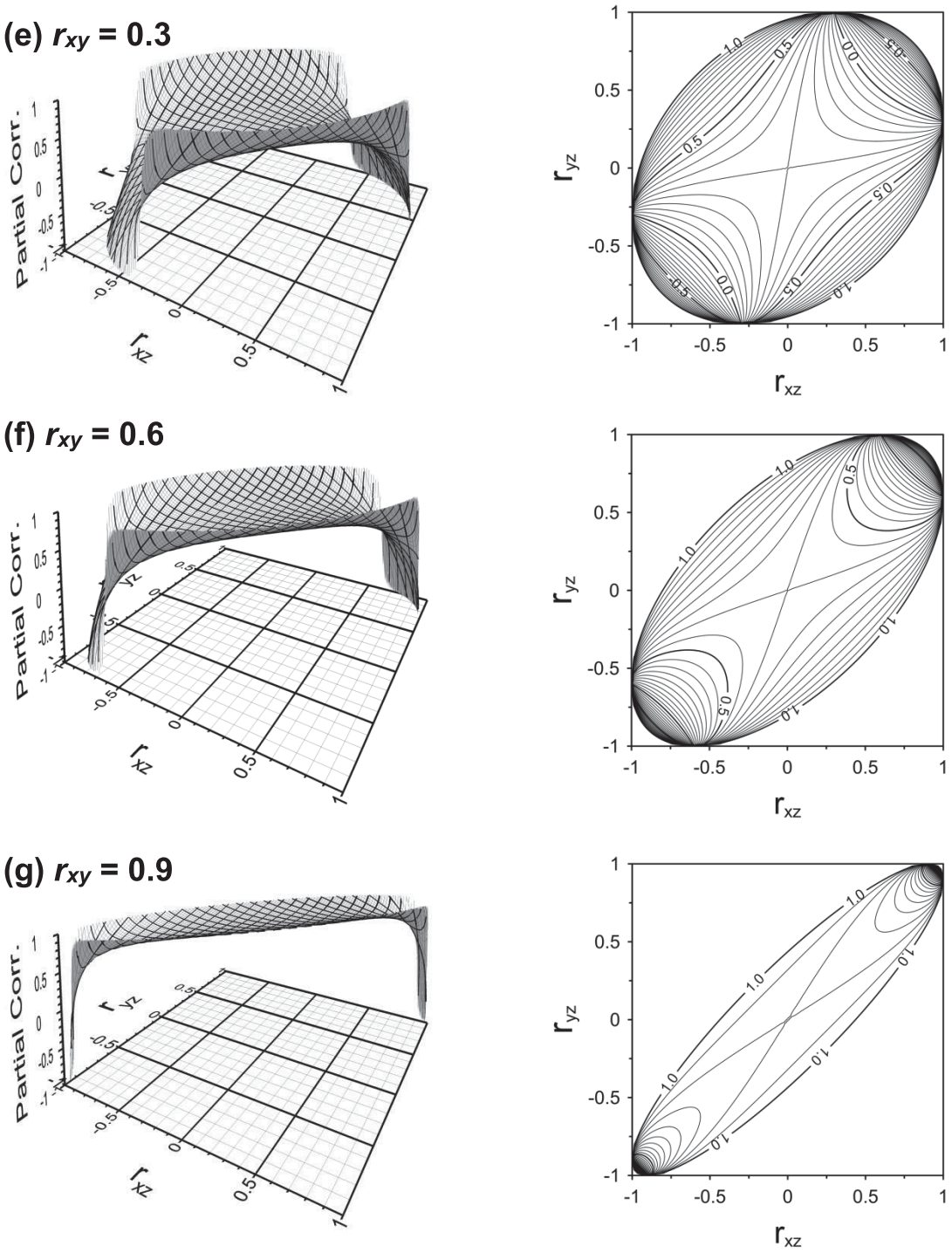
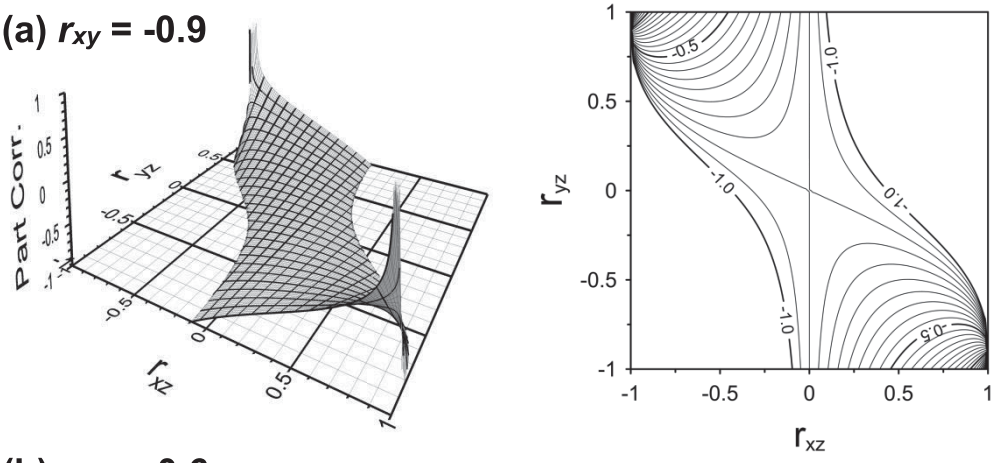
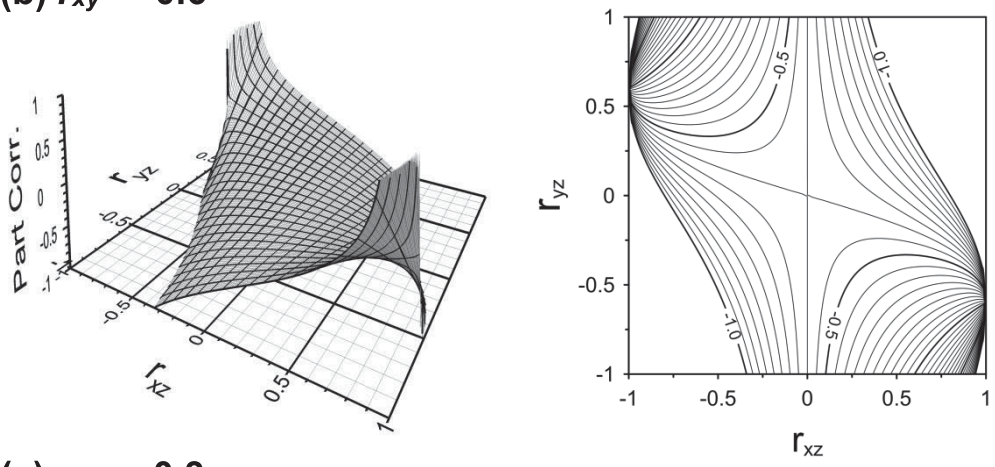


Figure 1. Surface maps (left column) and contour maps (right column) of a partial correlation coefficient. r_{xy}, r_{xz}, and r_{yz} are the correlation coefficients between x and y, x and z, and y and z, respectively.

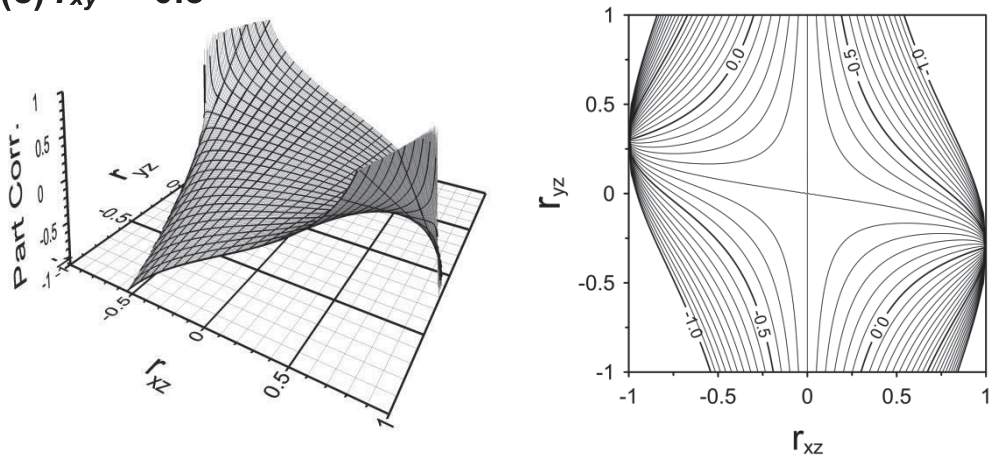
(a) r_{xy} = -0.9



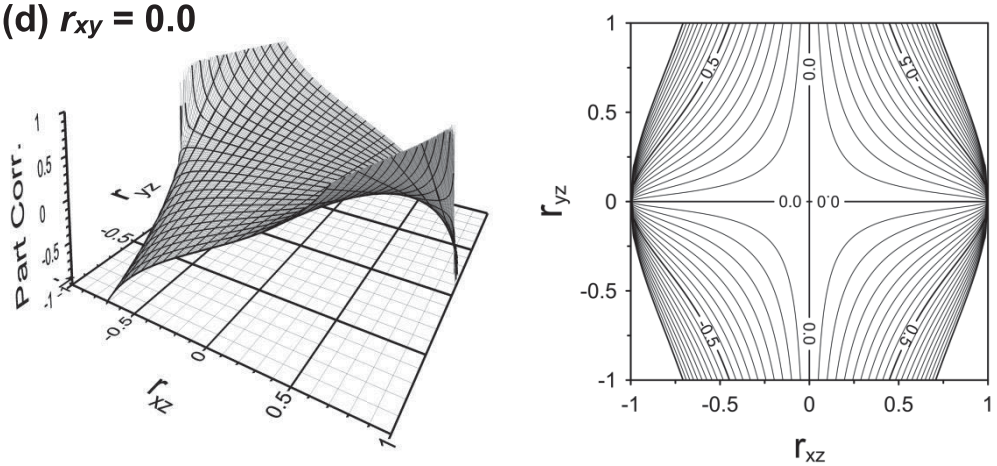
(b) r_{xy} = -0.6



(c) r_{xy} = -0.3



(d) r_{xy} = 0.0



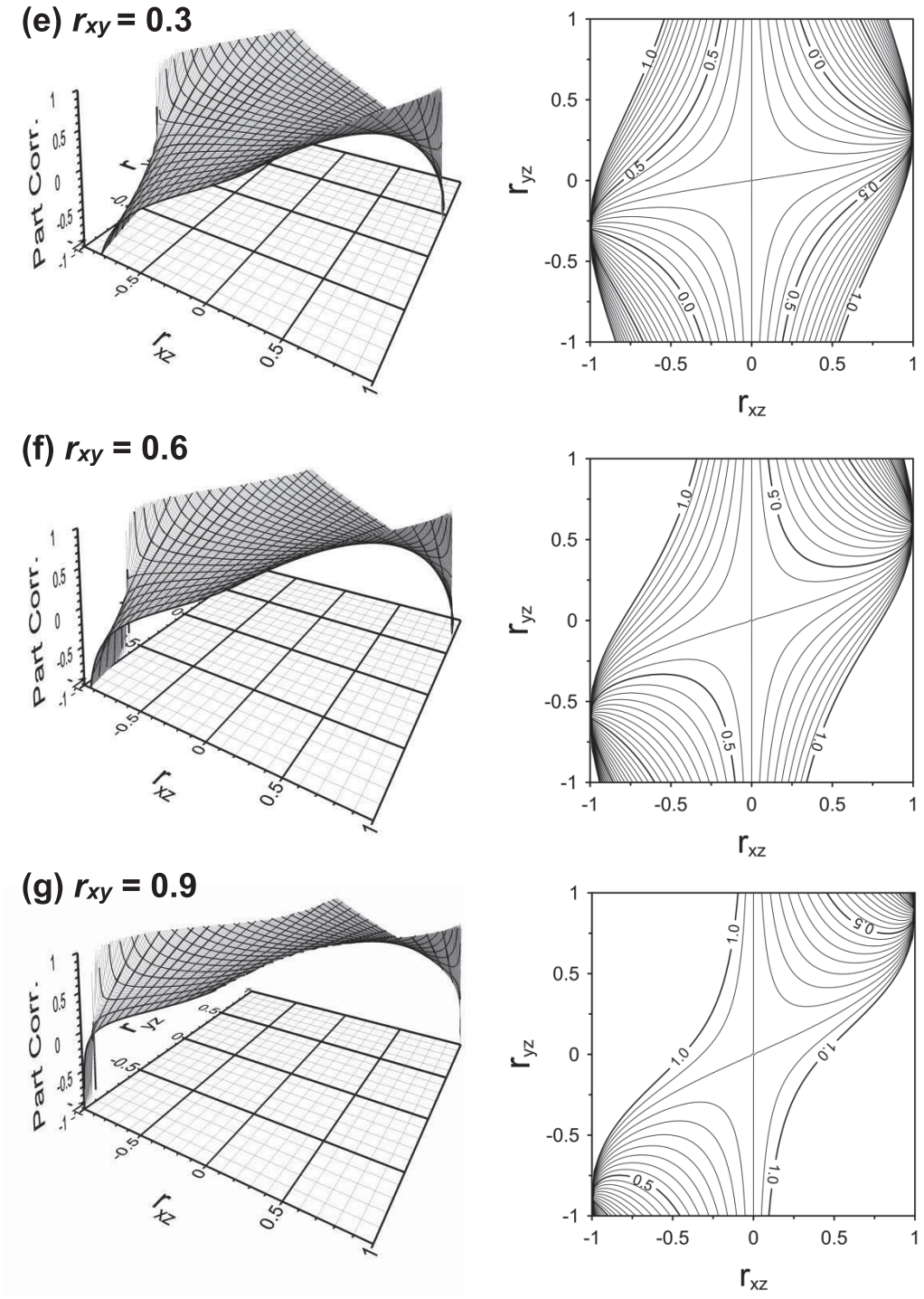


Figure 2. Surface maps (left column) and contour maps (right column) of a part correlation coefficient. r_{xy}, r_{xz}, and r_{yz} are the correlation coefficients between x and y, x and z, and y and z, respectively.

Figure 1 and 2. That is, the curves of partial and part correlation coefficients pass through similar points when r_{yz} goes to zero, but different points when r_{yz} goes to 1 or -1. Figure 1 and 2 should help to develop a concrete intuition of the correlations, and to achieve an understanding of the similarities and differences between the coefficients.

The surface and contour maps were drawn as a function of r_{xz} and r_{yz} by fixing r_{xy} at -0.9, -0.6, -0.3, 0.0, 0.3, 0.6, and 0.9 in this study. However, they are one of possible representations of the coefficients. The coefficients can be plotted as a function of r_{xy} and r_{xz} by fixing r_{yz}, or as a function of r_{xy} and r_{yz} by fixing or r_{xz}. Such plots also help to understand the characteristics of the coefficients.

This study has covered partial and part correlations involving three variables (Equations 1 and 2). These are the simplest forms of the partial and part correlations. These correlations can be extended to general forms, in which multiple variables z_1, z_2, ..., z_n are removed from x and/or y (cf. Anderson, 2003). Visualization of the correlations in the general forms remains an issue for future study.

5. References

Anderson, T. W. (2003). *An introduction to multivariate statistical analysis* (3rd ed.). New Dehli: Wiley India.

Davoudi, M., & Chavosh, M. (2016). The relationship between multiple intelligences and listening self-efficacy among Iranian EFL learners. *English Language Teaching*, *9*(6), 199–212.

Ebisu, A. M., & Iwanaga, M. (2016). The effects of contingent self-worth and rumination about failure on maladaptive perfectionists' motivation in academic situations. *The Japanese Journal of Personality*, *24*(3), 190–201. (In Japanese with English abstract)

Fichten, C. S., Amsel, R., Jorgensen, M., Nguyen, M. N., Budd, J., Havel, A., ... Asuncion, J. (2016). Theory of planned behavior: Sensitivity and specificity in predicting graduation and drop-out among college and university students. *International Journal of Learning, Teaching and Educational Research*, *15*(7), 38–52.

Gilden, D. L., & Wilson, S. G. (1995). Streaks in skilled performance. *Psychonomic Bulletin & Review*, *2*(2), 260–265.

Kashiwagi, K., & Hirayama, J. (2003). Marital norm, reality and satisfaction in middle-aged couples: Why are wives less satisfied than husbands? *The Japanese Journal of Psychology*, *74*(2), 122–130. (In Japanese with English abstract)

Kajabadi, P., HajiMohammadi, R., & Pahlavani, P. (2016). The relationship between EFL learners' resilience and autonomous learning. *International Journal of Applied Linguistics & English Literature*, *5*(4), 163–167.

Sasaki, J., Sugawara, K., & Tanno, Y. (2005). Why is the diverse U letter relationship between embarrassment and psychological distance observed?: In view of Schlenker and Leary (1982)'s self-presentation model of social anxiety. *The Japanese Journal of Psychology*, *76*(5), 445–452. (In Japanese with English abstract)

Segerstrom, S. C., Taylor, S. E., Kemeny, M. E., & Fahey, J. L. (1998). Optimism is associated with mood, coping, and immune change in response to stress. *Journal of Personality and Social Psychology*, *74*(6), 1646–1655.

Shibata, S. (2016). Development of the feelings toward nature scale and relationship between feelings toward nature and proximity to nature. *The Japanese Journal of Psychology*, *87*(1), 50–59. (In Japanese with English abstract)

Shimotsukasa, T., & Oshio, A. (2016). Structure and characteristics of entitlement: Focus on three dimensions of entitlement. *The Japanese Journal of Personality*, *24*(3), 179–189. (In Japanese with English abstract)

Thorsteinsson, E. B., Brown, R. F., & Richards, C. (2014). The relationship between work-stress, psychological stress and staff health and work outcomes in office workers. *Psychology*, *5*, 1301–1311.

Ueyama, R., & Sugimura, S. (2015). Relation between preschool teachers' experience, frequency of reflection, and perception of practical skills. *Japanese Journal of Educational Psychology*, *63*, 401–411. (In Japanese with English abstract)

Yoshida, T., & Murayama, K. (2013). Why do students often fail to use learning strategies that experts have found effective? An intra-individual analysis. *Japanese Journal of Educational Psychology*, *61*, 32–43. (In Japanese with English abstract)

Walberg, H. J., & Tsai, Shiow-Ling. (1983). Matthew effects in education. *American Educational Research Journal*, *20*(3), 359–373.